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Superconducting order in low-dimensional boson-fermion model: absence of finite-temperature transition

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Abstract. We present the application of the McBryan and Spencer method to a model describing a mixture of hard-core bosons and fermions interacting each others *via* a hybridization coupling. We prove upper bounds for some correlation functions at finite temperature and then rule out the possibility of long-range order of superconducting type in one and two dimensions.

PACS. 64.60.-i General studies of phase transitions – 74.20.Mn Nonconventional mechanisms (spin fluctuations, polarons and bipolarons, resonating valence bond model, anyon mechanism, marginal Fermi liquid, Luttinger liquid, etc. – 71.30.+h Metal insulator transitions and other electronic transitions

1 Introduction

One of the central issues in the field of statistical mechanics is the quest for criteria for the existence of phase transitions in physical systems. This issue can usually be addressed by the identification of a quantity, the order parameter, whose average vanishes on one side of the transition, but takes on a finite value on the other side. In a continuous phase transition, the order parameter may gradually evolve from zero at the critical point to a finite value on one side of the transition. Usually this happens on the low-temperature side of the transition. Obviously, for different kinds of phases, different order parameters must be chosen. The occurrence of a phase transition is often related to the failure of one of the phases to exhibit a certain symmetry property of the underlying Hamiltonians. In this respect, Bogoliubov has devised a method based on some inequalities for describing the occurrence of spontaneous symmetry-breaking in terms of quasi-averages [1]. The Bogoliubov inequality is a rigorous relation between two essentially arbitrary operators and the Hamiltonian of a physical system. A survey of existing proofs of the absence of finite-temperature phase transitions in low-dimensional systems based on this inequality is reported in [2].

Alternatively, one can calculate some two-point correlation functions and prove upper bounds for these functions. If these bounds decay exponentially or with power law, one rigorously rules out the possibility of a transition in an ordered state. The proof is based on a method developed by McBryan and Spencer [3] for classical spin systems, and its extension to quantum spin systems [4]. In these papers, the global continuous symmetry of the spin space plays an essential role. More recently, Koma and Tasaki [5] applying this method ruled out the possibility of condensation of singlet electron pairs, such as Cooper pairs or the η pairs [6], and the magnetic ordering in a general class of Hubbard models. These results hold in one and two dimensions at finite temperature. The class of Hubbard model they refer to corresponds to tight-binding models on one-dimensional lattice or planar lattice with hopping matrix term t_{xy} vanishing when $|x-y|$ exceed a finite constant R and less of another constant t when R is greater than $|x-y|$; the interaction term is an arbitrary function of the number operators. Therefore, the class of Hamiltonians includes for instance the Hubbard model, the periodic Anderson model, the $t-J$ model and also models containing long-range, random or spin-dependent interactions.

In this paper, by using the McBryan-Spencer approach we give some upper bounds for certain correlation functions that rule out the possibility of long-range superconducting pairing order for a the boson-fermion Hamiltonian widely used to describe the pseudogap phase in high-T_c superconductors [7]. The strategy here is to make use of the global U(1) symmetry related to the quantum mechanical phase with no further assumptions on the symmetry of the system since the U(1) symmetry exists in any quantum particle systems.

The rest of the paper is organized as follows. In Section 2 the boson-fermion model is introduced and its symmetry properties are discussed. The upper bounds for pairing correlation functions are obtained in Section 3 and finally, in Section 4 a summary of the results is presented.

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2 The model

The experimental evidence of a pseudogap together with numerous indications suggest that possibly a two-component system composed out of itinerant electrons and tightly bound electron-pairs (hardcore bosons) is involved in the high-T_c superconductivity. The clear experimental evidence of this pseudogap rules out any BCS type mechanism for superconductivity which would exclusively be controlled by amplitude fluctuations of the order parameter [8]. The onset of superconductivity at the critical temperature is controlled by phase fluctuations of the order parameter persisting to well above T_c and disappearing only above some characteristic temperature $T^\star$ where the electrons are no longer paired.

From a theoretical point of view these features can be addressed without having to resort to a specific microscopic mechanism for electron pairing and can be studied within effective models such as the boson-fermion model. This model was first introduced in connection with the many polaron problem in order to describe the crossover regime between adiabatic and nonadiabatic behavior [9]. In such a scenario bipolarons (bosons) are envisaged to coexist with quasifree fermions and an exchange coupling between the bosons and the fermions is assumed by which the bosons can decay into pairs of itinerant fermions and *vice versa*. The boson-fermion model is described by the following Hamiltonian:

$$H = H_D + H_t + H_\delta + H_g \,, \tag{1}$$

where

$$H_D = (D-\mu)\sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma}\,,$$

$$H_t = \sum_{i,j,\sigma} t(R_i - R_j) c^\dagger_{i\sigma} c_{j\sigma}\,,$$

$$H_\delta = (\delta - 2\mu)\sum_i b^\dagger_i b_i\,,$$

$$H_g = g\sum_i \left(b^\dagger_i c_{i\downarrow} c_{i\uparrow} + b_i c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} \right) .$$

Here $c^\dagger_{i\sigma}$ are the creation fermion operators referring to itinerant electrons with spin σ at i site and $b^\dagger_i$ are the creation operators of hard core bosons describing tightly bound electrons pair. The bare hopping integral for the electrons located at $\mathbf{R}_i$ and $\mathbf{R}_j$ is given by $t(R_i - R_j)$ and we assume that satisfies the reflection symmetry $t(R_i - R_j){=}t(R_j - R_i)$ and survives only for short-ranged overlapping. The bare electronic half bandwidth is denoted by D, the boson energy level by δ and the boson-fermion pair-exchange coupling constant by g. The chemical potential μ is common to fermions and bosons, up to a factor 2 for the bosons in order to guarantee the charge conservation.

Now we will discuss the symmetry properties of the model Hamiltonian (1). Let us define locally the following operators:

$$J^+_i = c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} - b^\dagger_i\,,$$
$$J^-_i = c_{i\downarrow} c_{i\uparrow} - b_i\,,$$
$$J^z_i = \frac{1}{2}\left(n_{i\uparrow} + n_{i\downarrow} + 2b^\dagger_i b_i - 2 \right) ,$$

where all the operators refer to the same lattice site.

It is straightforward to verify that these operator define a local SU(2) pseudospin symmetry, *i.e.*:

$$[J^\pm_i, J^z_j] = \mp J^\pm_i \delta_{ij}\,,$$

$$[J^+_i, J^-_j] = 2J^z_i \delta_{ij}\,.$$

3 Correlation functions

To demonstrate the results claimed in the Introduction, we replace the infinite lattice with a finite lattice of linear dimension L with periodic boundary conditions. The thermal expectation value of an arbitrary operator O is defined by $\langle O\rangle_L = \mathrm{Tr}\,[O\exp(-\beta H)]\,/\mathrm{Tr}\,[\exp(-\beta H)]$ where the trace is calculated over all the states. We then consider the infinite-volume expectation value defined by $\langle O\rangle = \lim_{L\to\infty}\langle O\rangle_L$.

To calculate the bound for some correlation functions we make use of the global quantum mechanical phase symmetry. The $U(1)$ gauge transformation is represented by the unitary operator:

$$U(\vartheta) = \mathit{\Pi}_{i\sigma}\exp\left[-\mathrm{i}\vartheta_i n_{i\sigma}\right]\exp\left[-2\mathrm{i}\vartheta_i N_i\right] , \tag{2}$$

where $\vartheta{=}\{\vartheta_i\}$ is a real arbitrary function on the lattice; $n_{i\sigma}$ (N_i) is the number operator for the electrons (bosons), respectively. The operator U is invertible and we have

$$\mathrm{Tr}\left[O\mathrm{e}^{-\beta H}\right] = \mathrm{Tr}\left\{U(\vartheta)OU(\vartheta)^{-1}\mathrm{e}^{-\beta[U(\vartheta)HU(\vartheta)^{-1}]}\right\}, \tag{3}$$

where the transformed Hamiltonian $U(\vartheta)H)U(\vartheta)^{-1}$ is

$$\widetilde{H} = H_D + \widetilde{H_t} + H_\delta + H_g\,, \tag{4}$$

with $\widetilde{H_t}$ given by

$$\widetilde{H_t} = \sum_{i,j,\sigma} t(R_i - R_j)\mathrm{e}^{-\mathrm{i}(\vartheta_i - \vartheta_j)} c^\dagger_{i\sigma} c_{j\sigma}\,.$$

Let us fix the lattice sites $\mathbf{R}_i$ and $\mathbf{R}_j$ and consider the following operator $O{=}J^+_i J^-_j$. Introducing a set of real functions ω_i which will be specified later, we have:

$$U(-\mathrm{i}\omega)OU(-\mathrm{i}\omega)^{-1} = \exp\left[-2(\omega_i - \omega_j)\right]O\,, \tag{5}$$

$$U(-\mathrm{i}\omega)HU(-\mathrm{i}\omega)^{-1} = H + X + \mathrm{i}Y\,. \tag{6}$$

Here

$$X = \sum_{n,m,\sigma} t(R_n - R_m)[\cosh(\omega_n - \omega_m) - 1]c^\dagger_{n\sigma} c_{m\sigma}\,,$$

$$Y = -\mathrm{i}\sum_{n,m,\sigma} t(R_n - R_m)\sinh(\omega_n - \omega_m) c^\dagger_{n\sigma} c_{m\sigma}\,,$$

are Hermitian matrices.

Following the same procedure used in reference [5], we can bound the equation (3) in this way:

$$\left|\mathrm{Tr}\left\{U(-\mathrm{i}\omega)OU(-\mathrm{i}\omega)^{-1}\mathrm{e}^{-\beta[U(-\mathrm{i}\omega)H)U(-\mathrm{i}\omega)^{-1}]}\right\}\right| \leq \mathrm{e}^{-2(\omega_i-\omega_j)} \parallel \mathrm{e}^{-\beta X} \parallel \mathrm{Tr}\left[\mathrm{e}^{-\beta H}\right] . \quad (7)$$

We have indicated with $\parallel W \parallel$ the maximum among the absolute values of the eigenvalues of the Hermitian matrix W.

With an appropriate choice of the functions $\{\omega_i\}$ we can give an upper bound of the left hand side of equation (7) in one and two dimensions. Making use of equation (3) and equation (7) and taking advantage of the results reported in reference [5], we can write

$$\mid \langle O\rangle_L \mid = \mathrm{Tr}\left[O\exp(-\beta H)\right]/\mathrm{Tr}\left[\exp(-\beta H)\right] \leq F(\mid R_i - R_j \mid), \quad (8)$$

where:

$$F(R_i - R_j) = \mathrm{e}^{[-f(\beta)k_1|R_i-R_j|]} \qquad (D=1) \quad (9)$$

$$F(R_i - R_j) = \mid R_i - R_j \mid^{-k_2 f(\beta)} \qquad (D=2) . \quad (10)$$

The function $f(\beta)$ is a decreasing function of β and k_i are finite constants.

Let us introduce now the following operators η_i^+ and η_i^-:

$$\eta_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + b_i^\dagger ,$$

$$\eta_i^- = c_{i\downarrow} c_{i\uparrow} + b_i .$$

These two operators and J_i^z above defined form another SU(2) local algebra.

If we fix the lattice sites $\mathbf{R}_i$ and $\mathbf{R}_j$ and take the following operator $\widetilde{O}=\eta_i^+\eta_j^-$, we can perform the same unitary transformation represented by $U(\vartheta)$ and we can prove the same bound as in equation (9) in the exactly the same manner, *i.e.*

$$\mid \langle \widetilde{O}\rangle_L \mid = \mathrm{Tr}\left[\widetilde{O}\exp(-\beta H)\right]/\mathrm{Tr}\left[\exp(-\beta H)\right] \leq F(\mid R_i - R_j \mid) . \quad (11)$$

Using equation (8) and equation (11), we get:

$$\mid \langle O\rangle_L + \langle \widetilde{O}\rangle_L \mid \leq \mid \langle O\rangle_L \mid + \mid \langle \widetilde{O}\rangle_L \mid \leq 2F(\mid R_i - R_j \mid) . \quad (12)$$

From this equation, we also deduce:

$$\mid c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{j\downarrow} c_{j\uparrow} + b_i^\dagger b_j \mid \leq F(\mid R_i - R_j \mid) . \quad (13)$$

The above bound rigorously rules out the possibility of the condensation of electrons or electron pairs as well as hard core bosons. Indeed, the pairing order parameter $\Delta \equiv\mid c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{j\downarrow} c_{j\uparrow} + b_i^\dagger b_j \mid$ is the sum of the order parameters for pairing of electrons and hard core bosons [10,11]. When Δ=0, separately the two order parameters vanish; this is a consequence of the hybridization term H_g that couples the two subsystems of particle and forces the two type of carriers to have the same critical temperature [11,12]. In other words, we can rule out the existence of condensation both for electrons and bosons.

4 Conclusions

In this paper, we have applied the McBryan and Spencer method to a model describing a mixture of localized pairs of electrons (bosons) and quasi-free electrons (fermions) hybridizing with each other *via* a charge exchange term. Firstly, we have considered a finite-system thermal average and then we have applied a unitary transformation which preserves the phase invariance of the model. We have shown that the boson-fermion model cannot support a superconducting long-range order, and this result holds for any non-zero temperature and any-filling in low dimensions. It is worth pointing out that in this paper for the first time the McBryan and Spencer approach has been applied to a mixed boson-fermion model. Furthermore, many results concerning the absence of finite-temperature phase transitions in low-dimensional systems are based on the application of the Bogoliubov inequality [2]. The general idea is to use this inequality to find an upper bound for the order parameter in question. The upper bound will normally depend on the external fields that couple to the order parameter, and on the order parameter itself. To find a phase transition to a state with a non-zero value of the order parameter, one must consider the behavior of the upper bound in the case of vanishing external field: if the assumption of non-zero order parameter leads to a violation of the upper bound, it must be dropped. Then, the only conclusion is that the order parameter vanishes and no phase transition occurs. Therefore, once a many-body model has been specified by its Hamiltonian, one must carefully chose the operators entering the Bogoliubov inequality so as to give the desired order parameter. On the other hand the application of McBryan and Spencer method is based on the use of the U(1) symmetry. The astonishing generality of this approach can be regarded as a demonstration of the fact that the electron hopping plays a fundamental role in condensation phenomena in itinerant electron systems and in this respect the results here presented are rather generic to analogous many-body models in one and two dimensions. It is worth stressing that the J operators introduced in Section 2 allow to deduce many interesting properties of the model. Namely, it can be shown that there are eigenstates of the Hamiltonian (1) supporting ODLRO and in some cases CDW too [13].

Finally, it must be stressed that other than two-point, long-range correlations are not excluded by the method here used. For instance in the isotropic Heisenberg model the four-site spin-spin correlation decays exponentially with increasing distance but a more complicate correlation displays long-range behavior [14]. Another example is the topological order as it occurs in the XY model, due to Kosterlitz-Thouless transition. This kind of order is not ruled out by an extension of the arguments presented in this paper to the XY model. It is possible that, below a finite temperature transition, the susceptibility diverges without the occurrence of a spontaneous magnetization because the correlations decay according to a power law [15].

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